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The possible influence of the de Broglie momentum—wavelength relation on plastic strain "autowave" phenomena in "active materials"

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Abstract

Certain very slow quasi-static tensile test results by Soviet investigators show that the resulting plastic deformation occurs as a wave process. An analysis of these data reveals a possible connection with the de Broglie momentum–wavelength relation, $V_D = h/(m\lambda)$. A link seems to exist between a macroscopic observable (wavelength) and the velocity (moving clamp speed) of atoms on a microscopic scale. Certain strain wave data from creep tests of aluminum were modeled via the running wave quantum relation for a particle (atom) in a one-dimensional box or line segment equal to the specimen length. These results have important implications with regard to plastic deformation theory. Published by Elsevier Science Ltd.

Keywords: de Broglie effect; Auto waves; Active materials; Plastic waves

1. Introduction

Over a decade has passed since Panin et al. (1988, 1989) first noticed and called attention to anomalous strain and rotation waves that were created during extremely slow motion tensile tests of metallic specimen undergoing plastic deformation. Recent reviews of similar test results for different metallic alloys are contained in Zuev et al. (1994, 1995) and Zuev and Danilov (1997b, 1998). "Autowaves" and/or "self-organization" of an "active medium" are terms used in these references to describe the mysterious strain waves observed via speckle holography interferometry instrumentation.

Zuev et al. (1995) state that, in nearly all cases, the wavelength (λ) magnitude is essentially 5–10 mm and the wave propagation velocity (V_{PV}) is of the order of 10^{-5}–10^{-4} m s^{-1}. Most of the tensile testing apparently utilized an Instron-1185 machine with a moving clamp velocity (V_{MC}) of approximately 1.6×10^{-6} m s^{-1}. The velocity of the atoms in the test specimen should range from 0 to V_{MC}.

The purpose of the present article is to suggest that there is some correlation with these experimentally observed wavelengths (λ) and the moving clamp velocity (V_{MC}) via the de Broglie momentum–wavelength relation that is

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$$V_D = h/(m\lambda). \tag{1}$$

In this famous relation, the symbols are defined as follows: h is the Planck's constant, 6.6262×10^{-27} $\text{g}\,\text{cm}^2\,\text{s}^{-1}$, m, mass of particle or atom (g), λ, wavelength (cm), V_D, de Broglie velocity, velocity of particle or atom ($\text{cm}\,\text{s}^{-1}$).

When λ is the experimentally observed wavelength and m, the mass of an atom (or m_{AV}, the average mass of an atom) in the test specimen, then V_D is of the same order of magnitude as V_{MC}, the moving clamp velocity. As noted above, the velocity of atoms in the test specimen should be of the same order of magnitude as V_{MC}. Quantitative comparative results are presented in Section 3.

Thus there appears to be a connection between Eq. (1) that holds for all particles (Moore, 1962; Daniels and Alberty, 1966) and the experimentally observed wave phenomena of very low speed tensile tests. That is, the observed wave phenomena are possibly the manifestation of a quantum effect (Eq. (1)) in addition to other possible material effects.

2. Background information

2.1. Quasi-static tensile tests

As noted in Section 1, Soviet investigators have performed a series of very slow speed tensile tests of various materials that have been deformed just past the elastic yield point and into the plastic regime. An Instron-1185 testing machine was employed with one end of the material specimen clamped and held fixed while the other end of the specimen was being pulled by a moving clamp whose velocity (V_{MC}) was nominally $100\ \mu\text{m}\,\text{m}^{-1}$ or $1.67 \times 10^{-6}\ \text{m}\,\text{s}^{-1}$ (Frolov et al., 1990; Danilov et al., 1991b; Zuev et al., 1991).

A rather sophisticated procedure was employed to extract strain information from the specimen in the standard tensile test arrangements described above. Holographic interferometry as discussed by Zuev et al. (1990) Panin et al. (1988), Frolov et al. (1990), and Zuev et al. (1997), was utilized to extract longitudinal (u) and transverse (v) particle displacement information from the specimen during the test. Numerical differentiation of these displacements with respect to the longitudinal (x) and transverse (y) directions yields $\partial u/\partial x, \partial u/\partial y, \partial v/\partial x$, and $\partial v/\partial y$. The longitudinal elongation strain is defined as $\varepsilon_{xx} = \partial u/\partial x$ and the transverse narrowing strain is $\varepsilon_{yy} = \partial v/\partial y$. The shear deformation is $\varepsilon_{xy} = \varepsilon_{yx} = (\partial v/\partial x + \partial u/\partial y)/2$ and the rotation is defined as $\omega_z = (\partial v/\partial x - \partial u/\partial y)/2$. These definitions are used in all cited documents containing experimental data.

When these strains (ε_{xx}, ε_{yy}, ε_{xy}) or rotations (ω_z) are plotted as a function of the longitudinal position (x) along the specimen, the resulting curves are oscillatory with a wavelength of several millimeters. The propagation velocity (V_{PV}) of these waves is on the order of 10^{-5}–$10^{-4}\ \text{m}\,\text{s}^{-1}$. This is at least an order of magnitude greater than the moving clamp velocity, V_{MC} (Danilov et al., 1990).

The occurrence of these plastic waves was mysterious and initially unexpected such that Panin et al. (1989) candidly asks why do they originate and how do they propagate. Several years later, these phenomena are still the source of considerable speculative discussion and analysis (Zuev and Danilov, 1997a,b; 1998; Zuev et al., 1997).

2.2. The particle momentum wave concept

Soon after their discovery, Panin (1990) cites sources of experimental evidence that demonstrate beyond doubt the wave nature of plastic deformation in quasi-statically stressed solids. This observation is commensurate with the particle momentum wave (PMW) concept of nonelastic motion in solids as suggested by Fitzgerald (1966a,b).

The basic PMW postulates are

1. The macroscopic deformation of solids under load is the integrated result of "free-field" atom motion.
2. These atoms travel through the solid as PMW subject to the rules of wave motion and are not governed exclusively by material structural characteristics.
3. The wavelength, λ, of these PMW is determined by the momentum (mV) via the de Broglie relation, Eq. (1).

Item 3 is a very important constraint which can apparently manifest itself enough to be evident in certain macroscopic test data (Billingsley, 1998a,b). Some examples of this will be briefly described. They are

1. Resonance frequencies in structural compliance testing.
2. Acoustic emission (sounds) during structural testing.
3. Hugoniot elastic limit (HEL) particle velocity decay to the $V_1 = h/(2md_1)$ level.

Several references, documenting the first successful calculations/comparisons of a macroscopic observable quantity, involving Planck's constant (h), during ordinary mechanical testing are cited by Billingsley (1998a). Resonance frequencies occurring during vibrational compliance testing of aluminum, stretched natural rubber (carbon atoms) and human cancellous (degenerate) bone segments (calcium and phosphorous atoms) could only be computed via the following relation for standing waves in a one-dimensional box or line segment:

$$v_{mn} = \left(\frac{h}{8ms^2}\right)n^2 \quad \text{standing waves,} \tag{2}$$

where $n = 1, 2, 3, \ldots, N/9$ and s, the segment length, $Nd \approx 10^{-4}$ cm (Fitzgerald, 1966a: pp. 38–42). The resonance frequencies from the bone tests were so sensitive to the correct mass of the calcium atoms that the mass for a calcium *isotope* produced better agreement with certain data. Thus, a quantum effect, usually associated with microscopic events, was revealed during macroscopic tests.

Note, that if Eq. (2) is multiplied by h, it is equivalent to the energy of a particle in a one-dimensional box or line segment which comes from a text book example of a solution for the Schroedinger wave equation as given in Daniels and Alberty (1966), Wark (1977), and Lowe (1993).

Fitzgerald (1966a, Chapter II) cites numerous references containing experimental data for acoustic emission from a variety of stressed materials (steel, aluminum, zinc, copper, lead, sodium chloride, and wood). He considers this as evidence of PMW where both standing and running waves can occur simultaneously. Discrete frequencies for the standing waves are given by Eq. (2), while the companion relation for running waves is Eq. (3) as written below with the same notation

$$v_{mn} = \left(\frac{h}{2ms^2}\right)n^2 \quad \text{running waves.} \tag{3}$$

Eqs. (2) and (3) are for low frequency modes and are derived from more general relations (Eqs. (2.26) and (2.27), respectively) in Fitzgerald (1966a, Chapter II). Note that $s = Nd$ is the segment length or long-range periodicity which depends on specimen size, grain size (GS), etc. and may be difficult to determine precisely. However, in many cases, as cited by Fitzgerald (1966a, Chapter II), s is on the order of 10^{-4} cm. In Section 3.1.2, Eq. (3) is applied where $s = L = 50$ mm $= 5$ cm, which is the specimen length.

Billingsley (1998a,b) provided comparative evidence to indicate that when λ was less than $2d_1$, where d_1 is the shortest (or average) distance between atoms, then the HEL precursor wave particle velocity ($U_{p_{HEL}}$) decays with travel distance (x) until $U_{p_{HEL}} = V_1 = h/(2md_1)$. *This is because λ cannot be less than $2d_1$ or an unstable situation develops.* This point is strongly emphasized by Fitzgerald (1966a,b). The wavelengths (via

Eq. (1)) in the HEL tests were on the order of 5 Å $= 5 \times 10^{-7}$ mm $= 5 \times 10^{-8}$ cm $= 5 \times 10^{-10}$ m. These wavelengths are well within the microscopic regime but apparently influence observations of $U_{p_{HEL}}$ on the macroscopic level. That is, when $U_{p_{HEL}}$ is greater than V_1, it decays asymptotically with travel distance to the V_1 level. When $U_{p_{HEL}}$ is greater than V_1, then λ is less than $2d_1$. When $U_{p_{HEL}}$ is less then V_1, then $U_{p_{HEL}}$ does not decay with travel distance because λ (via Eq. (1)) is greater than $2d_1$, which is a stable situation. This is another example where a microscopic quantum effect has apparently influenced macroscopic test results.

Balankin (1989) and Cherepanov et al. (1995) also mentioned a connection between the de Broglie relation and the HEL. Some of their ideas may also be appropriate to the subject matter (quasi-static tensile tests, QSTT data) of the present article.

As noted in Section 1, the wavelengths revealed from the quasi-static test data were on the order of 5–10 mm (5–10 $\times 10^7$ Å $= 0.5$–1.0 cm $= 5$–10×10^{-3} m). Thus they are about seven orders of magnitude larger than the lengths associated with the HEL phenomena. In the HEL tests, the relatively short wavelengths ($<2d_1$) created an unstable situation within the test specimen.

On the other hand, for the QSTT, the very low particle velocity ($\approx V_{MC}$) and the corresponding long wavelengths (via Eq. (1)) may also create unstable situations. The required wavelengths commensurate with particle velocities similar to V_{MC} via Eq. (1), may be incompatible with specimen size limitations, grain boundaries, crack locations, and voids within the specimen. If Eq. (1) cannot be locally satisfied within the specimen for some reason, then an unstable situation will exist. The more inhomogeneous a specimen is, the more likely that satisfaction of Eq. (1) will be inhibited. Restraints on the wavelength may dictate changes in particle velocity in order to satisfy Eq. (1). The observed rotations, ω_z, may be a result of this instability. It is suggested that the requirement to satisfy Eq. (1) (or Eqs. (2) or (3)) may be the reason for the surprisingly long wavelengths observed for certain strain waves which seem to be inexplicable otherwise. Because the specimen all have rectangular cross sections, the appropriate relation for standing wave frequencies may be

$$v = \frac{h}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right), \tag{4}$$

where n_x, n_y, n_z are the independent quantum numbers (1, 2, 3, 4, etc.) in each x, y, z direction. L_x, L_y, and L_z are the length, width, and thickness of the specimen, respectively. Eq. (4) comes directly from a textbook (Lowe, 1993) solution of the Schroedinger equation for a rectangular box. Even when L_x, L_y, and L_z are set or fixed for a given specimen, numerous discrete values of v are possible via n_x, n_y, and n_z independent combinations. The wavelengths will, of course, be related to specimen dimensions (or GS) which is commensurate with results from the QSTT data.

Both standing and running strain waves have been observed in the QSTT data by Zuev et al. (1995) and Zuev and Danilov (1997b) which is compatible with remarks by Fitzgerald (1996a, Chapter II) as mentioned previously.

Quantitative comparisons of the QSTT strain wavelength computations via Eqs. (1) and (3) are contained in Section 3.

3. Comparative analysis for five metallic materials

Table 1 provides a summary of the experimental data that were gleaned from the cited references. Wavelength, λ, data from ε_{xx} results were preferred because it involved only the displacement along the x-axis in the direction of the moving clamp velocity, V_{MC}.

Table 1
QSTT results for *active materials*

Materials	References	Specimen size (mm)			GS (mm)	Strain rate, $\dot{\varepsilon} \times 10^{-5}$ (s^{-1})	$V_{MC} \times 10^{-6}$ $L \times \dot{\varepsilon}$ ($m\,s^{-1}$)	$V_{PV} \times 10^{-5}$ ($m\,s^{-1}$)	λ (mm)	Remarks
		L	*W*	*T*						
Fe + 3%Si	Panin et al. (1998)	80	10	0.2	$\geqslant 10$	2	1.6	–	5 ± 2	$\lambda \approx GS/2 \approx W/2$
	Panin et al. (1989)	–	–	–	$\geqslant 10$	3	–	–	5 ± 2	$\lambda \approx GS/2$
	Panin (1990)	–	–	–	10	–	1.7	1.5	5 ± 2	$\lambda \approx 0.6$ GS
	Frolov et al. (1990)	50	10	0.3	10	3	1.7	1.5	5 ± 2	$\lambda \approx GS/2 \approx W/2$
		50	10	1.5						
$Fe_{40}Ni_{40}B_{20}$	Danilov et al., 1990	30	5	0.03	–	5	1.5	3.3	4 (ε_{xy}) (Fig. 1)	$\lambda \approx 0.8\ W$
		30	10	0.03	–	5	1.5	–	8 (ε_{xy}) (Fig. 2)	$\lambda \approx 0.8\ W$
		30	10	0.03	–	5	1.5	–	15 (ε_{xy}) (Fig. 4)	$\lambda \approx L/2$
	Frolov et al., 1990	30	5	0.025	–	–	1.7	8.0		
		30	10	0.025	–	–				
$Fe_{86}B_{14}$	Danilov et al. 1991b	50	6	0.024	–	3.3	1.8	6	18 (ε_{xy}) $\varepsilon_f = 0.6\%$	Both loading and unloading
		50	6	0.030	–			to 15	7 (ε_{xy}) $\varepsilon_f = 1.2\%$	Both loading and unloading
$Cu_{84}Ni_{10}Sn_6$	Zuev et al. (1994)	25	5	1	–	6.7	1.675	(6.5 ± 0.3)	3.5 (ε_{xy}) or 4.0 (Fig. 1)	Easy glide stage
								(7.5 ± 0.4)	6.0 (ε_{xx}) (Fig. 1) 8.0 (Text)	Linear hardening stage
Aluminum	Frolov et al. (1990)	50	10	0.3	10	–	1.66	–	20–24 (ε_{xy})	*Fig.* 7 } *Coarse grain*
		50	10	1.5	(coarse)				5–10 (ω_z)	*Fig.* 7 } *Coarse grain*
		50	10	1.5	0.04 (fine)	–	1.66	–	5 (ε_{xy})	*Fig.* 8 } *Fine grain*
		50	10	0.3	0.03 (fine)				4 (ω_z)	*Fig.* 8 } *Fine grain*
									5 (ε_{xy})	*Fig.* 8 } *Fine grain*
									3 (ω_z)	*Fig.* 8 } *Fine grain*
	Danilov et al. (1991a)	50	10	1.5	0.04	3.4	1.7	1.5	5.55	$\lambda = 13.6 + 2.5\ Ln$ (GS)
					1.00				13.6	Eq. (4) for ε_{xy} wave lengths in Danilov et al. (1991a)
					10.00				19.35	

However, λ from ε_{xy} and ω_z results were also used for comparative purposes, particularly if ε_{xx} data were not given (plotted or discussed). ε_{yy}, ε_{xy}, and ω_z are functions of the displacements (v) in the lateral direction, perpendicular to V_{MC}. So their wavelengths may be more dependent on the specimen width (W) or thickness (T) than ε_{xx} wavelengths. Commensurate with this line of thought, ε_{xx} wavelengths may be more dependent on the specimen length.

The Soviet researchers also noticed some strain wavelength dependence on grain size GS. This is commensurate with standing or running waves being contained in (or confined to) a box (grain boundaries).

In references where V_{MC}, $\dot{\varepsilon}$ (overall strain rate), and L (specimen length) were all given, it was found that they agreed with the following relation:

$$V_{\mathrm{MC}} = L \cdot \dot{\varepsilon}. \tag{5}$$

This implies that

$$\varepsilon = \frac{\Delta L}{L} = \frac{u}{L} = \text{gross or total overall strain}, \tag{6}$$

$$\dot{\varepsilon} = \frac{\varepsilon}{\Delta t} = \left(\frac{\Delta L}{\Delta t}\right)\frac{1}{L} = \frac{\dot{u}}{L} = \frac{U_{\mathrm{p}}}{L} = \text{gross or total overall strain rate}, \tag{7}$$

$$u = \text{displacement of a point in the } x\text{-direction},$$

$$U_{\mathrm{p}} = \frac{\Delta L}{\Delta t} = \dot{u} = \text{gross or total overall particle velocity} = V_{\mathrm{MC}}.$$

So V_{MC} is, at least, a measure of particle (atom) velocity, U_{p}. Consequently, V_{D}, as computed from Eq. (1) from the experimental strain or rotation wavelengths, is compared to V_{MC}. For references where V_{MC} is not given, but $\dot{\varepsilon}$ and L are given, then V_{MC} is computed from Eq. (5). Wavelength (λ) data, from references that did not state V_{MC} or provide enough information($\dot{\varepsilon}$ and L) to compute it, were not utilized in the quantitative comparisons contained in Table 2.

Zuev et al. (1991, Fig. 4) presents analytical/calculated results for the distribution of ε_{xx} along the x-axis of a specimen undergoing QSTT. Additional details and results (such as u and U_{p} distributions) would be of considerable interest.

3.1. Aluminum

3.1.1. Quasi-static tensile tests data

Polycrystalline fine grain and coarse grain aluminum was tested via the QSTT technique with speckle holography instrumentation and the results were reported in Frolov et al. (1990); Danilov et al. (1991a), and Zuev et al. (1995). Specimen sizes, GS, and wavelengths from these sources are listed in Table 1. Note that there is a dependence of wavelength on GS; however, the wavelength may also be influenced by the specimen size to some extent via Eqs. (2) and (3), or (4), or the running wave counterpart of Eq. (4).

V_{D} was computed for a range of values corresponding to the experimental wavelengths from Frolov et al. (1990) and Danilov et al. (1991a). V_{D} either brackets V_{MC}, or is close enough to V_{MC}, such that V_{D} and V_{MC} are the same order of magnitude.

3.1.2. Creep test data

Information contained in Danilov et al. (1991c) documents creep tests performed on macrocrystalline (10 mm GS) 99.7% pure aluminum specimen where speckle holography was employed to acquire strain

Table 2
de Broglie velocity comparison with QSTT results

Materials	Experimental				$V_D = h/(m\lambda)$			Remarks
	References	$V_{MC} \times 10^{-6}$ (m s^{-1})	λ (mm)	$V_{PV} \times 10^{-5}$ (m s^{-1})	$m \times 10^{-23}$ g	λ (mm)	$V_D \times 10^{-6}$ (m s^{-1})	
Fe + 3%Si	Panin (1990)	1.7	5 ± 2	1.5	9.273 ($= m_{fe}$)	3.0	2.382	
						4.0	1.786	$V_D \approx V_{MC}$
	Frolov et al. (1990)	1.67	5 ± 2	1.5		5.0	1.429	$V_D \approx V_{MC}$
						6.0	1.141	
						7.0	1.021	
$Fe_{40}Ni_{40}B_{20}$	Danilov et al. (1990)	1.53	4–8	3.3	7.967 ($= m_{AV}$)	4.0	2.08	
			15			8.0	1.04	V_D brackets V_{MC}
						15.0	0.555	
$Fe_{86}B_{14}$	Danilov et al. (1991b)	1.8	7–18	6–15	8.226 ($= m_{AV}$)	7.0	1.15	$V_D = 64\%\ V_{MC}$
						18.0	0.49	$V_D = 25\%\ V_{MC}$
$Cu_{84}Ni_{10}Sn_6$	Zuev et al. (1994)	1.675	3.5–8.0	7.5	11.02 ($= m_{AV}$)	3.5	1.718	$V_D \approx V_{MC}$
						4.0	1.503	$V_D \approx V_{MC}$
						6.0	1.002	$V_D \approx 0.60\%\ V_{MC}$
						8.0	0.750	$V_D \approx 0.44\%\ V_{MC}$
Aluminum	Frolov et al. (1990)	1.67	4–25	–	4.48 ($= m_{AL}$)	4	3.697	
						8	1.849	$V_D \approx V_{MC}$
						9	1.643	$V_D \approx V_{MC}$
						17	0.870	$V_D \approx V_{MC}/2$
						25	0.592	
	Danilov et al. (1991a)	1.7	5.55	1.5		5.55	2.665	V_D Brackets V_{MC}
			13.60			13.60	1.088	
			19.35			19.35	0.764	

displacement data. These creep tests differed from the QSTT procedure in that constant tensile load (or stress) was applied to a specimen instead of a constant moving clamp velocity. The specimen working size was $L = 50$ mm, $W = 10$ mm, and $T = 1.5$ mm.

In the creep tests, the tensile load (or stress) above the elastic limit was varied from one test specimen to another so that the strain rate, $\dot{\varepsilon}$, also varied. Thus, a variety of different strain rates were achieved during these tests that were conducted at 300 K (27°C or 80.6°F). Analysis of the displacement data revealed the wave nature of the strain (ε_{xy}) and rotation (ω_z) similar to what had been observed in QSTT data. The wave like appearance of these data is displayed in Fig. 1 of Danilov et al. (1991c).

Danilov et al. (1991c, Table 1) provided a summary of their results which lists, λ, θ, and V_{PV} for a variety of strain rates, $\dot{\varepsilon}$, where $\dot{\varepsilon}$ is constant during the steady state creep stage. Note that

$$\theta = \frac{1}{v} = \text{period of one wave cycle (s)} \tag{8}$$

and according to Danilov et al. (1991c)

$$V_{\mathrm{PV}} = \frac{\lambda}{\theta} = \lambda v = \text{wave propagation velocity,} \tag{9}$$

where v is the wave frequency (s^{-1}).

An overall gross value of the particle velocity, U_{p}, was computed via Eq. (7), so that

$$U_{\mathrm{p}} = L \cdot \dot{\varepsilon}. \tag{10}$$

Values of the wavelengths, recorded in Table 1 of Danilov, et al. (1991c) ranged from (5 ± 1) to (7 ± 1) mm. Thus essentially, λ was not affected as the strain or creep rate, $\dot{\varepsilon}$, was varied from 3.16×10^{-6} to $20.62 \times 10^{-6}\ \mathrm{s}^{-1}$.

These wavelengths are somewhat smaller than those (10 or 11 mm) that can be obtained from measuring Δx distances between "peak-to-peak" ε_{xy} and ω_z amplitudes in Fig. 1(a) and (b), respectively, of Danilov et al. (1991c). However, their magnitudes are still on the order of GS (GS $\approx$ 10 mm) and specimen width ($W = 10$ mm). In fact, these ε_{xy} and ω_z wavelengths may be related to the specimen length ($L = 50$ mm) as pointed out in the following quantum related analysis.

Exploratory computations for running waves in aluminum were made via Eq. (3) with the segment lengths, s, equal to L, the specimen length in the x (tensile load) direction. According to Lowe (1993, p. 32), the allowed wavelengths are

$$\lambda = \frac{2s}{n}, \tag{11}$$

where n is the same integer as defined for Eq. (2) and n was chosen so that λ was equal to, or close to, certain values of λ (4, 5, 6, and 7 mm) measured by Danilov et al. (1991c). When Eq. (11) is multiplied by Eq. (3), in accordance with Eq. (9), the result for the wave propagation velocity, V_{PV}, is

$$V_{\mathrm{PV}} = \frac{hn}{ms}. \tag{12}$$

Also, when Eq. (11) for λ is substituted in to Eq. (1), the de Broglie particle velocity is

$$V_{\mathrm{D}} = \frac{hn}{2ms} \tag{13}$$

$$V_{\mathrm{D}} = \frac{V_{\mathrm{PV}}}{2}. \tag{14}$$

Thus, there is a simple theoretical relation between V_{D} and V_{PV} for running waves in a line segment.

Table 3 lists values of λ, v, (or θ) and V_{PV} data from the six different creep experiments (different $\dot{\varepsilon}$) and also lists for comparative purposes, values of λ, v, (or θ) and V_{PV} from Eqs. (11), (3), (8), and (12), respectively. For experiments 1–4, with $s = L$, n could be selected such that:

(a) λ calculated $\approx \lambda$ (experimental),
(b) v calculated $\approx v$ (experimental),
(c) V_{PV} calculated $\approx V_{PV}$ (experimental).

Note that when $\lambda \approx 5$ mm ($n = 20$), the comparison is remarkably close (less than 4.0% difference in θ and V_{PV}). The fact that V_{PV}, computed via Eq. (12), compares reasonably well with the experimental data is important since it provides some rationale for the magnitude of the observed values of V_{PV} (for the creep data and perhaps even for QSTT data as well) and the long wavelengths also.

For experiments 5 and 6, note that in order for θ (calculated) to compare favorably with θ (experimental), then λ (calculated $\approx$3 mm) must be about one-half λ (experimental $\approx$6 mm). For all tests, differences by as much as a factor of 10 or more exist between $U_p = L \cdot \dot{\varepsilon}$ (experiment) and V_D computed via Eq. (13). So $U_p = L \cdot \dot{\varepsilon}$ (experimental) may not apply to the creep test data. The Soviet investigators (Danilov et al., 1991c) did not use it in their data analysis.

Section 4 contains additional remarks about these comparison discrepancies.

3.2. Ferrosilicon (Fe + 3%Si)

QSTT results for a ferrosilicon alloy (Fe + 3%Si) are contained in Panin et al. (1988,1989) and Frolov et al. (1990). Table 1 provides pertinent information acquired from the cited references. Table 2 shows that the de Broglie velocities V_D (Eq. (1)) computed for strain (ε_{xy}) wavelengths from 3 to 7 mm are comparable to V_{MC} magnitudes.

Frolov et al. (1990) evaluated the period, $\theta \approx 300$ s, for the ferrosilicon waves, and with the experimental wavelengths, the wave propagation velocity, $V_{PV} \approx 1.5 \times 10^{-5}$ m s^{-1} was then computed via Eq. (9). V_{PV} is about an order of magnitude larger than V_{MC} which is on the order of the particle velocity.

Some exploratory computations were performed via the running wave relation, Eq. (3), for the ferrosilicon data. However, a consistent match with λ, θ (or v) and V_{PV} was not obtained.

3.3. Iron–nickel–boron alloy

Amorphous alloy $Fe_{40}Ni_{40}B_{20}$ QSTT plastic strain (ε_{xx}, ε_{xy}) and rotation (ω_z) data are presented and described primarily in Danilov et al. (1990) and are mentioned in Frolov et al. (1991b). Certain information about the specimen and test results is given in Table 1. Note that the specimen thickness, T, is 30 μm $= 30 \times 10^{-6}$ m $= 30 \times 10^{-6} \times 10^{3}$ mm $= 30 \times 10^{-3}$ mm $= 0.030$ mm. $Fe_{40}Ni_{40}B_{20}$ data discussed in Danilov et al. (1991b) are the same data that are the subject of Danilov et al. (1990) where the following wavelength data were found.

The wavelength of shear strain, ε_{xy}, data is about 4 mm for a specimen width of 5 mm (Fig. 1) and is about 8 mm (Fig. 2) for a specimen width of 10 mm. That is for ε_{xy}, the wavelength, λ, is about 0.8 W. However, for ε_{xx}, a wavelength of 15 mm can be obtained from Fig. 4 for a 10 mm wide specimen. This wavelength is 0.5 L where L is the length of the specimen (30 mm). These wavelengths (4, 8, and 15 mm) are substituted into Eq. (1) to compute an associated V_D. These values are given in Table 2 that indicates they bracket V_{MC} and are the same order of magnitude.

The experimental investigators (Danilov et al. 1990) determined an oscillation period, $\theta \approx 120$ s, and combined this with the observed wavelength ($\lambda \approx 4$ mm) to compute the wave propagation velocity ($V_{PV} \approx 3.3 \times 10^{-5}$ m s^{-1}). Again, V_{PV} is about an order of magnitude larger than V_{MC}.

Table 3
Comparison of aluminum creep test data with exploratory computational results

Source	Item	Units	Exp. 1	Exp. 2	Exp. 3	Exp. 4	Exp. 5	Exp. 6
Exp.	$\dot{\varepsilon}$	s^{-1}	3.16×10^{-6}	4.50×10^{-6}	4.60×10^{-6}	9.79×10^{-6}	13.67×10^{-6}	20.62×10^{-6}
Exp.	$\lambda =$ wave length	mm	6 ± 1	5 ± 1	7 ± 1	6 ± 1	6 ± 1	6 ± 1
Cal.	$\lambda = 2s/n$		6.667 ($n = 15$)	5.00 ($n = 20$)	6.25 ($n = 16$)	5.00 ($n = 20$)	3.703 ($n = 27$)	3.03 ($n = 33$)
	$s = L = 50$ mm							
Exp.	θ cycle period	s	1450	864	1270	820	450	300
Cal.	$\theta = 1/v$	s	1502.47	845.13	1320.56	845.13	463.72	310.43
Exp.	$v = 1/\theta$	s^{-1}	6.8966×10^{-4}	11.57×10^{-4}	7.874×10^{-4}	12.20×10^{-4}	22.22×10^{-4}	33.33×10^{-4}
Cal.	$v = hn^2/2ms^2$	s^{-1}	6.6557×10^{-4}	11.8325×10^{-4}	7.5727×10^{-4}	11.8325×10^{-4}	21.5647×10^{-4}	32.2137×10^{-4}
Exp.	$V_{PV} = \lambda/\theta = \lambda v$	$m\,s^{-1}$	4.8×10^{-6}	5.7×10^{-6}	5.6×10^{-6}	7.3×10^{-6}	13.3×10^{-6}	20.0×10^{-6}
Cal	$V_{PV} = hn/ms$	$m\,s^{-1}$	4.4374×10^{-6}	5.916×10^{-6}	4.7324×10^{-6}	5.916×10^{-6}	7.984×10^{-6}	9.7608×10^{-6}
Exp.	$U_p = L \cdot \dot{\varepsilon}$	$m\,s^{-1}$	0.158×10^{-6}	0.225×10^{-6}	0.230×10^{-6}	0.490×10^{-6}	0.684×10^{-6}	1.031×10^{-6}
Cal.	$V_D = hn/2ms$	$m\,s^{-1}$	2.2185×10^{-6}	2.9581×10^{-6}	2.3665×10^{-6}	2.9581×10^{-6}	3.9942×10^{-6}	4.8814×10^{-6}

Exp. = Experimental from Danilov et al. (1991c); Cal. = Calculated.

Some exploratory computations with Eq. (3) for running waves were made but a consistent match with λ, θ (or v), and V_{PV} was not accomplished.

3.4. Iron–boron alloy

QSTT results for another amorphous iron alloy ($Fe_{86}B_{14}$) are given in Danilov et al. (1991b). Table 1 lists pertinent information about the specimen, test conditions, and test results. Data for wavelengths of ε_{xy} were given as 18 mm at a total strain, $\varepsilon_f = 0.6\%$, and were the same for both loading and unloading. When the total strain, $\varepsilon_f = 12\%$, the wavelength was 7 mm for both loading and unloading. The corresponding V_D via Eq. (1) are from 25 to 64% of V_{MC} as shown in Table 2.

3.5. Copper–nickel–tin alloy

Face centered single crystals (FCSC) of a copper–nickel–tin alloy ($Cu_{84}N_{10}Sn_6$) were subjected to QSTT with speckle interferometry instrumentation. The test results are reported in Zuev et al. (1994) and certain portions of this information are listed in Table 1.

During this test, initially, a plastic strain wave, ε_{xx}, that is about 6–8 mm wide traveled from the fixed end of the specimen to the moving end held by the moving clamp. This was called the "stage of easy glide." Its propagation velocity, V_{PV}, was $(6.5 \pm 0.3) \times 10^{-5}$ m s^{-1} or (0.065 ± 0.003) mm s^{-1}. Near $t = 150$ s, oscillatory ε_{xx} waves of this stage had wavelengths of 3.5–4.0 mm as measured from Fig. 1 in the reference cited above. The corresponding de Broglie velocities, V_D, from Eq. (1) are 1.718×10^{-6} and 1.503×10^{-6} m s^{-1}, respectively. They bracket the moving clamp velocity ($V_{MC} \approx 1.675 \times 10^{-6}$ m s^{-1}) and are remarkably close to it. Perhaps this is because, at this time ($t \approx 150$ s), the wave front position ($x \approx 20$ mm) is close to the location of the moving clamp ($x \approx 25$ mm) where particle velocities, U_p, in the specimen should equal V_{MC}.

The magnitude of V_D corresponding to the initially observed wavelengths of 6–8 mm are 1.002×10^{-6} m s^{-1} ($0.60 V_{MC}$) and 0.750×10^{-6} m s^{-1} ($0.44 V_{MC}$), respectively, as given in Table 2. These early values for V_D (or U_p) seem reasonable near the fixed end of the specimen where $U_p = 0.0$.

When the "easy glide" plastic strain, ε_{xx}, wave front reached the specimen end at the moving clamp, it reversed direction and propagated toward the end held by the fixed clamp. This was called the "strain hardening stage" and it traveled with a velocity, V_{PV}, equal to $(7.5 \pm 0.4) \times 10^{-5}$ m s^{-1}. These ε_{xx} waves, called "seats of plastic flow," were shaped like the positive part of a sine wave plus essentially a flat portion near zero magnitude where the negative part of a sinusoidal wave would ordinarily be found. This is called a rectified half sine wave in Lathi (1965), p. 193, and 8 mm is the distance that separates these traveling waves. Considering 8 mm to be the wavelength yields V_D, via Eq. (1), to be 0.750×10^{-6} m s^{-1} that is 44% of V_{MC}.

This rectified half sine wave shape of the "strain hardening stage" ε_{xx} waves may be caused by interference or beating phenomena of two waves (easy glide waves and its reflection from the moving clamp location). This type of phenomena could also create standing waves such as shown for $Cu_{84}Ni_{10}Sn_6$ in Fig. 4(a) of Zuev et al. (1997b) and for aluminum single crystals in Fig. 5 of Zuev et al. (1995).

4. Conclusions

Quantitative exploratory computations with the de Broglie momentum–wavelength relation and comparisons with experimental QSTT wavelength data have been made for five metallic materials. These collected comparisons are listed in Table 2. The computed values of V_D via Eq. (1) with experimental λ data as input are close to, or bracket, V_{MC} (moving clamp velocity) that is a measure of the velocity of individual atoms in the system. This reasonably good agreement provides a plausible explanation for the long wavelengths experimentally observed from QSTT strain data via the speckle holography technique.

In addition, quantitative exploratory computations with the quantum related running wave relation (Eq. (3)) for a particle (atom) in a one-dimensional box (line segment) have been compared to creep test results for a polycrystalline aluminum specimen. These comparative results are presented in Table 3. For four of the six experiments (where $\dot{\varepsilon}$ was varied), it was shown that, with judiciously selected wavelength input via Eq. (11), then Eqs. (3) and (12) provide reasonably good predictions for the frequency, v (or θ) and V_{PV}, respectively.

In general, for both the QSTT and creep test speckle holography results, the de Broglie–Fitzgerald quantum related PMW concept appears to provide an answer, or at least provides a clue, to previously inexplicable behavior such as:

(a) The unusually long strain wavelengths.
(b) The magnitude of the wave propagation velocity, particularly for the creep tests.

This is considered to be correct even though the analytical/experimental result comparisons were not always one-to-one or otherwise consistent.

Some reasons for this disagreement are as follows:

(a) The application of Eqs. (1) and (3) to the observed strain (ε_{xx}, ε_{xy}) and rotation (ω_z) data instead of particle velocity (U_p or V_p) data is, of course, questionable. However, explicit data for U_p (x-direction) and V_p (y-direction) were not available. Thus, the analysis was applied to the strain/rotation data with the implicit assumption that the particle velocity wavelengths were somewhat similar.

(b) ε_{xy} and ω_z wavelength data were employed because ε_{xx} data were not always available. ε_{xx} wavelength data would have been preferable since only u (displacement in the x-direction) is involved. The questionable use of Eqs. (1) and (3) is somewhat more justifiable when applied to ε_{xx} data than when applied to ε_{xy} and ω_z data. This is because ε_{xx} does not explicitly involve the transverse displacement (y-direction) and the measured wavelengths in the cited references were generally in the x-direction where via V_{MC} and/or $\dot{\varepsilon}$, some estimate of U_p could be made.

Certain experiments (1–4) of the creep test (Danilov et al., 1991c) seemed to indicate that the V_{PV} magnitude was consistent with Eq. (12) from the running wave analysis. However, in these cases, the estimated $U_p = L \cdot \dot{\varepsilon}$ (experiment) results did not compare very well with the de Broglie velocity (Eq. (1) or Eq. (13)). Perhaps this is related to the discussion in reason (b).

The running wave analysis did not provide consistent results between λ, θ (or v), and V_{PV} for the QSTT ferrosilicon and iron–nickel–boron alloy data where a check with experimental data was possible. On the other hand, the de Broglie velocity (Eq. (1)) with the QSTT experimental strain/rotation wavelength input, compared very well with V_{MC} (a yardstick of the particle velocity, U_p, magnitude).

So some creep test data for aluminum were amenable to the running wave analysis but the QSTT data for two alloys (ferrosilicon and iron–nickel–boron) were not. This could be caused by inherent differences between QSTT and creep testing load application and other reasons (noted previously) as well.

The documented agreement between the PMW concept analysis results and the available experimental strain/rotation wavelength data may be fortuitous. However, it should be sufficient to motivate further experimental/theoretical work to verify, clarify, or refute this quantum related PMW application to exceptionally slow moving (quasi-static) test data. Some recommendations for additional work are contained in the following section.

The QSTT and creep test speckle holography strain wave data are a basic and, consequently, a very important contribution to the plasticity literature. Most of the interpretive analysis and discussion of the QSTT/creep speckle holography strain wave data by the Soviet investigators (Panin, 1990; Zuev et al., 1995;

Zuev and Danilov, 1997a,b) has been from a continuous media approach that attempts to bridge the microscopic, mesoscopic, and macroscopic regimes of deformation.

Section 2.2 of the present document cites three examples where quantum related PMW effects on the discrete particle (atom) level have shown up unexpectedly in macroscopic testing. The quantum PMW effect is rather subtle and is not an overpowering influence in the vast majority of macroscopic structural testing. Consequently, it is unexpected and, initially, anomalous when it occurs.

In addition, PMW influences are usually difficult to document conclusively because of experimental data deficiencies (Billingsley, 1998a). However, based upon the rather limited quantitative comparisons in the present document, the PMW effect is strongly suggested as the root cause of, at least, the otherwise inexplicable long wavelengths exhibited by the cited QSTT/creep test strain data.

5. Recommendations

Some suggestions for additional work are implicitly contained in the previous sections of this document. Some of these tasks could be accomplished more readily by the cited original Soviet investigators if the initial raw speckle holography data are still available. These tasks are collected here and explicitly delineated.

1. Compute U_p and V_p (particle or mass velocities) from the raw displacement (u and v) data, if possible. If these U_p, V_p data exhibit oscillatory wave behavior (with a wavelength) then the de Broglie relation can be applied without the caveats necessary when strain data are analyzed. Thus every effort should be made to acquire these particle velocity (U_p and V_p) data.

2. ε_{xx} strain wavelength information (not shown or published in some of the cited references) should be examined via the PMW concept as was done for ε_{xy} and ω_z data in the present document. As noted in Section 4, application of the PMW analysis to ε_{xx} data is much more justifiable than it is when applied to ε_{xy} and ω_z data.

3. All of the QSTT data documented in the cited references were taken with the moving clamp velocity, V_{MC}, equal to $(1.6 \pm 0.1)10^{-6}$ m s^{-1}. It is recommended that QSTT speckle holography data be acquired, if possible, where V_{MC} is at least a factor of 2 or 3 larger or smaller than 1.6×10^{-6} m s^{-1}. A factor of 10 or an order of magnitude change in V_{MC} would be preferable. This would allow a good check on the PMW application to be made since the strain wavelengths should vary in accordance with Eq. (1) when written as

$$\lambda = \frac{h}{mV_{MC}}. \tag{15}$$

4. An analysis of the creep testing procedure should be performed to ascertain if the gross overall particle or mass velocity is given by Eq. (10) which is: $U_p = L \cdot \dot{\varepsilon}$. U_p as given by Eq. (10) did not compare very favorably with V_D as given by Eq. (1) or Eq. (13) in the running wave analysis (Eq. (3)) of the creep data in Danilov et al. (1991c).

5. Some additional exploratory computations, via the PMW concept, should be performed on the data analyzed in the present document. This would specifically involve the two- and three-dimensional forms of the one-dimensional running wave relation (Eq. (3)). This would allow the lateral (width and/or thickness) dimensions of the specimen to be taken into account because strain wavelengths are influenced by specimen width.

6. Some consideration should be given to: What is the "proper" specimen length since it may extend past the fixed and moving clamp locations (working length) insofar as running waves are concerned? These uncertain end (or length) boundary conditions may be one reason the exploratory running wave computations (Eq. (3)) for the QSTT data were not successful (Sections 3.2 and 3.3).

References

Balankin, A.S., 1989. Synergetics and the Mechanics of a Deformable Body. Soviet Technical Physics Letters 15 (11), 878–880.

Billingsley, J.P., 1998a. The decay limit of the Hugoniot elastic limit. International Journal of Impact Engineering 21 (4), 267–281.

Billingsley, J.P., 1998b. The Hugoniot elastic limit decay limit. Proceedings of the Shock Compression of Condensed Matter 1997 Conference, AIP Publisher, Woodbury, NY, pp. 199–202.

Cherepanov, G.P., Balankin, A.S., Ivanova, V.S., 1995. Fractal fracture mechanics – a review. Engineering Fracture Mechanics 51 (6), 997–1033.

Daniels, F., Alberty, R.A., 1966. Physical Chemistry. Wiley, NY.

Danilov, V.I., Panin, V.Ye., Mnikh, N.M., Zuyev, L.B., 1990. Relaxation waves during plastic deformation of amorphous alloy $Fe_{40}Ni_{40}B_{20}$. Physics of Metals and Metallography 69 (6), 181–186.

Danilov, V.I., Zuyev, L.B., Mnikh, N.M., Panin, V.Ye., Shershova, L.V., 1991a. Wave effects during plastic flow of polycrystalline Al. Physics of Metals and Metallography 71 (3), 187–193.

Danilov, V.I., Yevsikov, S.V., Zuyev, L.B., 1991b. Deformation waves and microshear processes in amorphous alloys. Physics of Metals and Metallography 71 (5), 172–175.

Danilov, V.I., Yavorskii, A.A., Zuyev, L.B., Panin, V.E., 1991c. Wave phenomena during creep in macrocrystalline aluminum. Soviet Physics Journal 34 (4), 283–286.

Fitzgerald, E.R., 1966a. Particle Waves and Deformation in Crystalline Solids. Interscience Publishers, NY.

Fitzgerald, E.R., 1966b. Particle waves and audio frequency modes in crystals. Journal of Acoustics Society of America, Part 1, pp. 1856–1869.

Frolov, K.V., Panin, V.E., Zuev, L.B., Makutov, N.A., Danilov, V.I., Mnikh, N.M., 1990. Relaxation waves in plastic deformation. Soviet Physics Journal 33 (2), 110–123.

Lathi, B.P., 1965. Signals, Systems, and Communication. Wiley, NY.

Lowe, J.P., 1993. Quantum Chemistry, Second edition. Academic Press, London.

Moore, W.J., 1962. Physical Chemistry. Wiley, NY.

Panin, V.Ye., Zuyev, L.B., Danilov, V.I., Mnikh, N.M., 1988. Peculiarities of the displacement field during plastic deformation of large-grained silicon iron. Physics of Metals and Metallography 66 (5), 160–165.

Panin, V.Ye., Zuev, L.B., Danilov, V.I., Mnikh, N.M., 1989. Plastic deformation as a wave process. Soviet Physics Journal 33 (2), 99–110.

Panin, V.Y., 1990. Wave nature of plastic deformation in solids. Soviet Physics Journal 33, 99–110.

Wark, K., 1977. Thermodynamics. McGraw-Hill, NY.

Zuev, L.B., Danilov, V.I., Mnikh, N.M., 1990. Speckle-interferometer method of recording and analyzing displacement fields during plastic deformation. Industrial Laboratory 56 (2), 224–230.

Zuev, L.B., Panin, V.E., Mnikh, N.M., Danilov, V.I., Peker, N.V., 1991. Plastic deformation waves on a yield-point plateau. Soviet Physics Doklady 36 (4), 336–338.

Zuev, L.B., Danilov, V.I., Kartashova, N.V., 1994. Space–time self-organization of plastic deformation of FCC single crystals. JETP Letters 60 (7), 553–555.

Zuev, L.B., Danilov, V.I., Gorbatenko, V.V., 1995. Autowaves of localized plastic deformation. Technical Physics 40 (5), 456–462.

Zuev, L.B., Danilov, V.I., 1997a. The nature of large-scale correlations in plastic flow. Physics of the Solid State 39 (8), 1241–1245.

Zuev, L.B., Danilov, V.I., 1997b. Plastic deformation viewed as evolution of an active medium. International Journal of Solids and Structures 34 (29), 3795–3805.

Zuev, L.B., Danilov, V.I., Kartashova, N.V., Barnnikova, S.A., 1997. The self-excited wave nature of the instability and localization of plastic deformation. Materials Science and Engineering A: Structural Material: Properties Microstructure and Processing A234–236, 699–702.

Zuev, L.B., Danilov, V.I., 1998. Plastic deformation viewed as a self-excited wave process. Journal De Physique IV 8, 413–420.